توجد قوانين ل(Beta , Gamma , bessel)

تمت كتابتهم ولكنهم متواجدين بالمحاضرة ☺

سكشن رياضة

يوجد شرح لـ "Gamma" لكنه في المحاضرة.

$$\int_0^{\frac{1}{2}} x^{m-1} \ln\left(\frac{1}{2x}\right) dx = -\int_0^{\frac{1}{2}} x^{m-1} \ln(2x)\, dx$$

$\ln(2x) = -t$

$$-\int_\infty^0 \left(\frac{1}{2} e^{-t}\right)^{m-1} -t \cdot \frac{-1}{2} e^{-t}\, dt$$

$2x = e^{-t}$

$x = \frac{1}{2} e^{-t}$

$$\int_0^\infty \left(\frac{1}{2}\right)^{m-1} \left(e^{-t}\right)^{m-1} + t * \frac{1}{2} e^{-t}\, dt$$

$dx = \frac{-1}{2} e^{-t}$

$\ln(0) = -t$

$-\infty = -t$

$$\frac{1}{2^m} \int_0^\infty t^1 \cdot e^{-tm}\, dt$$

$y = tm$ ، $t = \frac{y}{m}$ ، $dt = \frac{1}{m} dy$

$$\frac{1}{2^m} \int_0^\infty \frac{y}{m} e^{-y} \cdot \frac{1}{m} dy = \frac{1}{2^m m} \int_0^\infty y \cdot e^{-y} dy$$

$$= \frac{1}{2^m m} \Gamma(2) = \frac{1}{m 2^m}$$

Beta fn

$\boxed{ex}$ $\int_0^1 x^5 (1-x)^3 \, dx$

$= B(6,4) = \dfrac{\Gamma(6)\,\Gamma(4)}{\Gamma(10)} = \dfrac{5!\,3!}{9!} = \ldots$

$\boxed{2}$ $\int_0^1 \dfrac{5x}{\sqrt{1-x^5}} \, dx = \int_0^1 5x \left(1-x^5\right)^{\frac{-1}{2}} dx$

$t = x^5 \longrightarrow x = t^{\frac{1}{5}} \longrightarrow dx = \frac{1}{5} t^{\frac{-4}{5}} dt$

$I = \int_0^1 5\, t^{\frac{1}{5}} (1-t)^{\frac{-1}{2}} \cdot \frac{1}{5} t^{\frac{-4}{5}} \, dt$

$= \int_0^1 t^{\frac{-3}{5}} (1-t)^{\frac{-1}{2}} dt = B\left(\frac{2}{5}, \frac{1}{2}\right) = \ldots$

$\boxed{3}$ $\int_0^{\frac{\pi}{2}} \sqrt{\dfrac{\sin^3\theta}{\cos\theta}} \, d\theta = \frac{1}{2} * 2 \int_0^{\frac{\pi}{2}} \sin^{\frac{3}{2}}\theta \, (\cos\theta)^{\frac{-1}{2}} \, d\theta$

$2m-1 = \frac{3}{2} \Rightarrow m = \frac{5}{4}$ & $2n-1 = \frac{-1}{2} \rightarrow n = \frac{1}{4}$

$I = \frac{1}{2} B\left(\frac{5}{4}, \frac{1}{4}\right)$